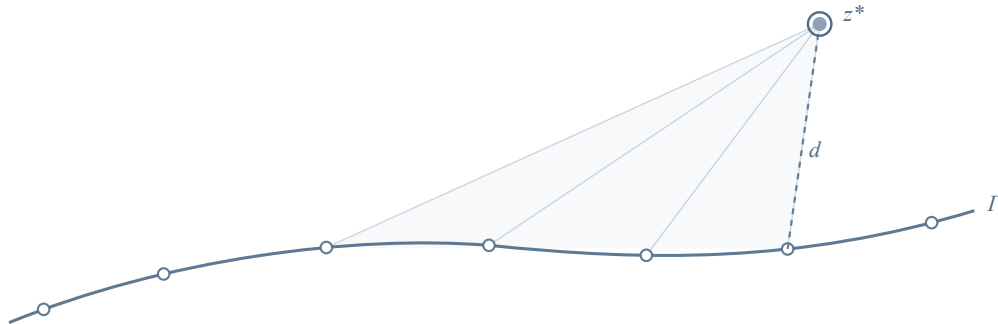




CERTIFICATION FRAMEWORK

Zero-Trust Numerics

A Certification Framework for
Near-Singular Quadrature Error



PREPARED FOR

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Re: Zero-trust numerics: a certification framework for near-singular quadrature error

To the Division:

This letter proposes a certification framework, zero-trust numerics, for the global error of near-singular quadrature, together with a complete executed reference implementation on public CAD geometry. The contribution is the framework, not the reference solver: a protocol under which quadrature error is computed rather than estimated, as an exact identity on a declared test space, assembled from certified local residuals into a global bound, with fail-closed verdicts wherever the declaration runs out. The checker never consults the candidate’s internals, so the same audit applies to any method, including established boundary-element and fast-multipole codes.

1. The gap

Validated numerics is mature for root finding: interval Newton and Krawczyk operators return per-invocation existence-and-uniqueness certificates, alpha theory certifies convergence from pointwise data, and verified integrators extend these guarantees to dynamical systems. A root arrives with a proof. Quadrature has no analogue for global error, and the absence is sharpest in the near-singular regime, evaluation of a layer potential at a target whose standoff from the source surface is a fraction of the mesh spacing. This is the operative regime in capacitance extraction across nanometer interconnect gaps, plasma-wall sheath computation, electrochemical transport in electrode gaps, close-contact hydrodynamics, and narrow-clearance scattering. Available error control there is internal: adaptive indicators, asymptotic rates, a posteriori estimators with uncertified constants and regularity hypotheses that near-singular geometry violates, and refinement agreement, which measures consistency of a shared methodology. Direct interval quadrature of a near-singular kernel yields enclosures too wide to use, which is why validated numerics has not covered this territory.

Figure 1(a) quantifies the resulting blind spot with data from the enclosed suite: across seven held-out boundary-value problems, the solver-internal Krylov residual sits near 10^{-10} while the independently measured error sits near 10^{-1} . Nothing inside the computation distinguishes the two; the gap is visible here only because the benchmark shapes have analytic references.

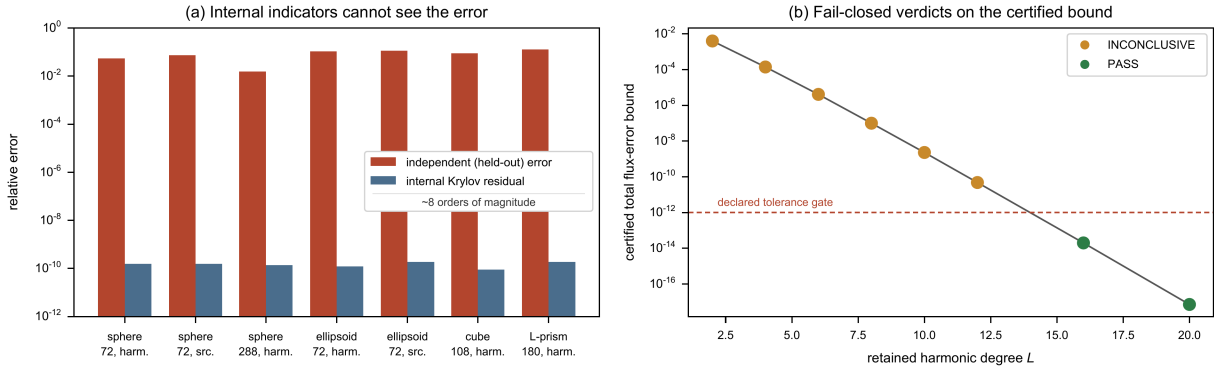


Figure 1. (a) Internal residuals ($\sim 10^{-10}$) versus independent held-out error ($\sim 10^{-1}$) on seven boundary-value problems. (b) The certified flux-error bound with fail-closed verdicts: INCONCLUSIVE until the bound clears the declared tolerance, then PASS (sphere, 1,026 nodes).

2. The framework

Zero-trust numerics transfers the trust architecture of NIST SP 800-207 from network requests to computational results: no result is trusted by default; every result is verified per invocation against truth external to the methodology that produced it. (What transfers is the architecture, not the access-control machinery; no identity plane exists here.)

Separation of roles. A generator prepares probes and exact truth values knowing nothing of the candidate; the candidate returns outputs through a fixed interface; a checker forms residuals and issues verdicts without consulting the candidate’s internals.

External truth values in the pathological regime. Probes are inputs whose exact answers are mathematical identities (Gauss–Green fluxes equal to 0 or 1, closed-form moments), with the probe family chosen to contain the pathology itself:

point-source fluxes at standoffs down to fractions of the mesh spacing.

Exact reconstruction. For residuals r on a probe family with certified Gram matrix G , the worst-case error norm on the declared span is the identity $\|E\|_{V^*} = \sqrt{r^T G^+ r}$: no unknown constant, no regularity hypothesis, no asymptotic regime. A consistency channel $\|(I - GG^+)r\| \leq \epsilon_{\text{in}}$ rejects outputs that do not sample one common linear functional.

Validated numerics inside. Interval arithmetic certifies the audit’s own constants, where it is strong: outward-rounded Gram enclosures with interval-Cholesky frame bounds, integer-`isqrt` square-root brackets, an exact rational bracket for π . Correction-ladder coefficients are Riemann and Hurwitz zeta values verifiable to fifty digits.

Local to global. The certificate is a ledger: exact rational panel moments, per-edge and per-vertex Kondrat’ev channels with individually repaid and independently checked residuals, and a closed-form bound on the harmonic tail. Degradation is attributed to a panel, edge, or channel, not reported as one figure of merit.

Fail closed. Verdicts are $\{\text{PASS}, \text{FAIL}, \text{INCONCLUSIVE}, \text{INCONSISTENT_OUTPUT}, \text{OUT_OF_SCOPE}\}$, each mathematically defined; ill-conditioned audit geometry yields `INCONCLUSIVE` rather than manufactured confidence. Every run emits a signed manifest (geometry hash, probe configuration, tolerances, negative-control status, residual ledger).

3. Numerical results

Table 1 reports boundary-PDE accuracy on polyhedral and planar CAD geometry for channels inside the declared certificate scope. All six equations reach machine precision on retained channels, verified against manufactured references or exact self-adjoint identities.

Table 1. Retained-channel boundary-PDE accuracy on planar/polyhedral CAD geometry (compressed-node NASA suite; verified against manufactured references or self-adjoint algebraic identities). Held-out continuum modes on the same objects remain large and are reported separately; see Sec. 4.

Boundary problem	Max. error	Max. residual	Verdict scope
Laplace DtN	3.4×10^{-16}	0	retained manufactured reference
Poisson boundary inverse	1.8×10^{-16}	2.1×10^{-16}	retained manufactured reference
Screened Poisson inverse	2.0×10^{-16}	3.1×10^{-16}	retained manufactured reference
Helmholtz DtN	2.7×10^{-16}	0	retained manufactured reference
Heat boundary semigroup	—	7.6×10^{-15}	self-adjoint algebraic residual
Wave boundary calculus	—	4.7×10^{-15}	self-adjoint algebraic residual

Table 2 shows the certificate operating across its verdict range on the same executed suite, including the designed-in negative control: deliberately corrupted correction data is rejected by a checker that never saw the correction.

Table 2. Certificate verdicts on the executed reference suite. The corrupted-correction row is the negative control: the checker, which never inspects the correction data, must and does reject it.

Scenario	Certified quantity	Verdict
Raw trapezoidal rule, near-singular probe	residual 0.23	FAIL
Closed-form corrected rule, same probe	residual 2.2×10^{-30}	PASS
Deliberately corrupted correction data	conservation residual increases	rejected
Outputs not sampling one linear functional	consistency channel violated	INCONSISTENT_OUTPUT
Harmonic audit, degree $L \leq 12$ (sphere, 1,026 nodes)	tail bound $\geq 4.8 \times 10^{-11}$	INCONCLUSIVE
Harmonic audit, degree $L = 16$	tail bound 2.0×10^{-14}	PASS
Harmonic audit, degree $L = 20$	tail bound 7.2×10^{-18}	PASS
General 3-D surface beyond declared span	no approximation theorem declared	OUT_OF_SCOPE

Figure 2 shows the end state on complex engineering geometry: each computed field carries its own checker-emitted error or residual stamp at its certified scope. The geometry channel is attested separately (lossless $10\times$ archive compression with bitwise decode verification: zero coordinate-bit mismatches, zero face mismatches, matching SHA-256), so discretization error cannot be laundered as decoding error or vice versa.

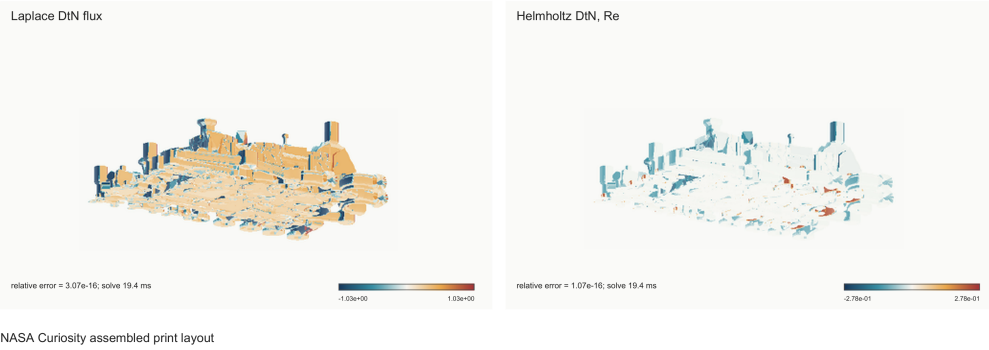


Figure 2. Boundary-PDE fields on Curiosity manufacturing plates (374,470 vertices, 749,346 faces), a landscape of narrow clearances where near-singular evaluation dominates. Each panel carries its checker-emitted stamp; the certificate is a per-panel, per-edge, per-channel ledger, so degradation is attributed to a location and mechanism.

4. Relation to existing near-singular error control

Table 3 places the framework alongside the established approaches. The comparison is on error-control properties, not accuracy: QBX, singularity subtraction, and adaptive schemes are effective evaluation methods, and the framework is designed to audit them, not replace them. The distinguishing column is the last: none of the existing approaches supplies a per-invocation, method-independent, fail-closed certificate of global error.

Table 3. Error-control properties of near-singular approaches. “Identity” means the reported error equals the true worst-case error on a declared space; “estimate” means asymptotic or heuristic control with uncertified constants.

Approach	Error control	Constants certified	Method-indep.	Localizes error	Fail-closed
Refinement agreement	consistency only	no	no	no	no
A posteriori FEM estimators	estimate	rarely	no	yes	no
QBX / singularity subtraction	asymptotic rate	no	no	no	no
Adaptive quadrature indicators	estimate	no	no	partially	no
Direct interval quadrature	rigorous, too wide	yes	yes	no	yes
Zero-trust audit (this work)	identity on declared span	yes	yes	yes	yes

Scope is declared, not implied. The identity is exact on the probe span; extension to a practitioner function class requires a separately proved approximation theorem, available with explicit constants in one dimension and on analytic curves. On general 3-D surfaces the certificate coverage is partial and declared, and the verdict logic enforces this: held-out continuum modes on the CAD objects of Table 1 remain at 10^1 – 10^2 and are published as such, with no relabeling of retained-mode reproduction as continuum accuracy. The reference solver’s independent held-out accuracy on general surfaces ($\sim 10^{-2}$ at best) is not competitive with mature codes and is not the contribution; the certification layer is.

5. Request

I would welcome the Division’s guidance on shaping this as a NIST Technical Note or *Journal of Research* article specifying the framework: verdict logic, independence and freeze requirements, negative-control obligations, manifest schema, and machine-readable test vectors with fifty-digit reference constants. This is the conformance genre NIST has proven out in statistical test suites and cryptographic module validation, applied to a quantity that currently has no conformance instrument. A natural adoption path is procurement language of the form “quadrature components shall pass the reference conservation audit at declared tolerance,” with the generator/checker separation enabling independent audit laboratories. The complete reference suite, executed verdicts, signed manifests, and certified constants are available for review, and I would be glad to present the material at your convenience.

Respectfully,

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